Cryptography

- Overview
- Symmetric Key Cryptography
- Public Key Cryptography
- Message integrity and digital signatures

Cryptography issues

Confidentiality: only sender, intended receiver should "understand" message contents

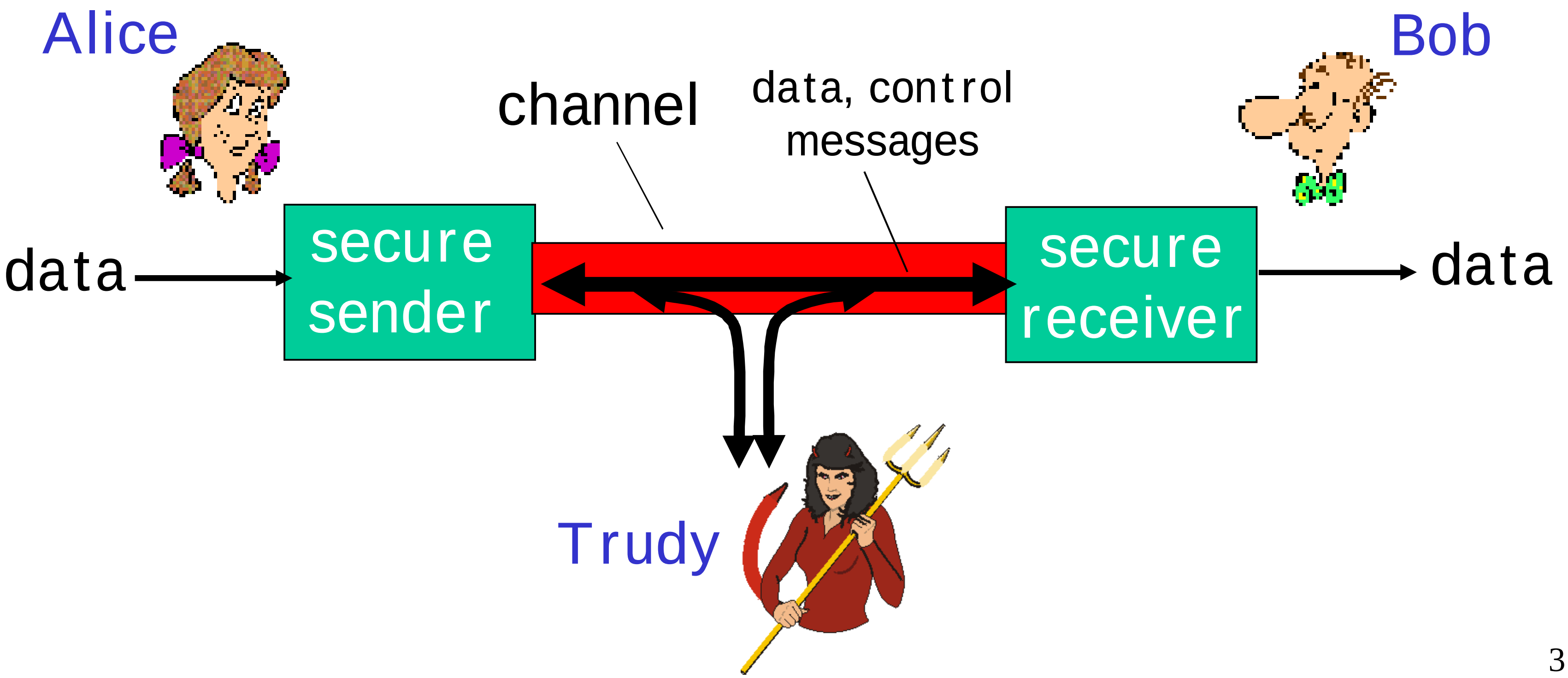
- sender encrypts message
- receiver decrypts message

End-Point Authentication: sender, receiver want to confirm identity of each other

Message Integrity: sender, receiver want to ensure message not altered (in transit, or afterwards) without detection

Friends and enemies: Alice, Bob, Trudy

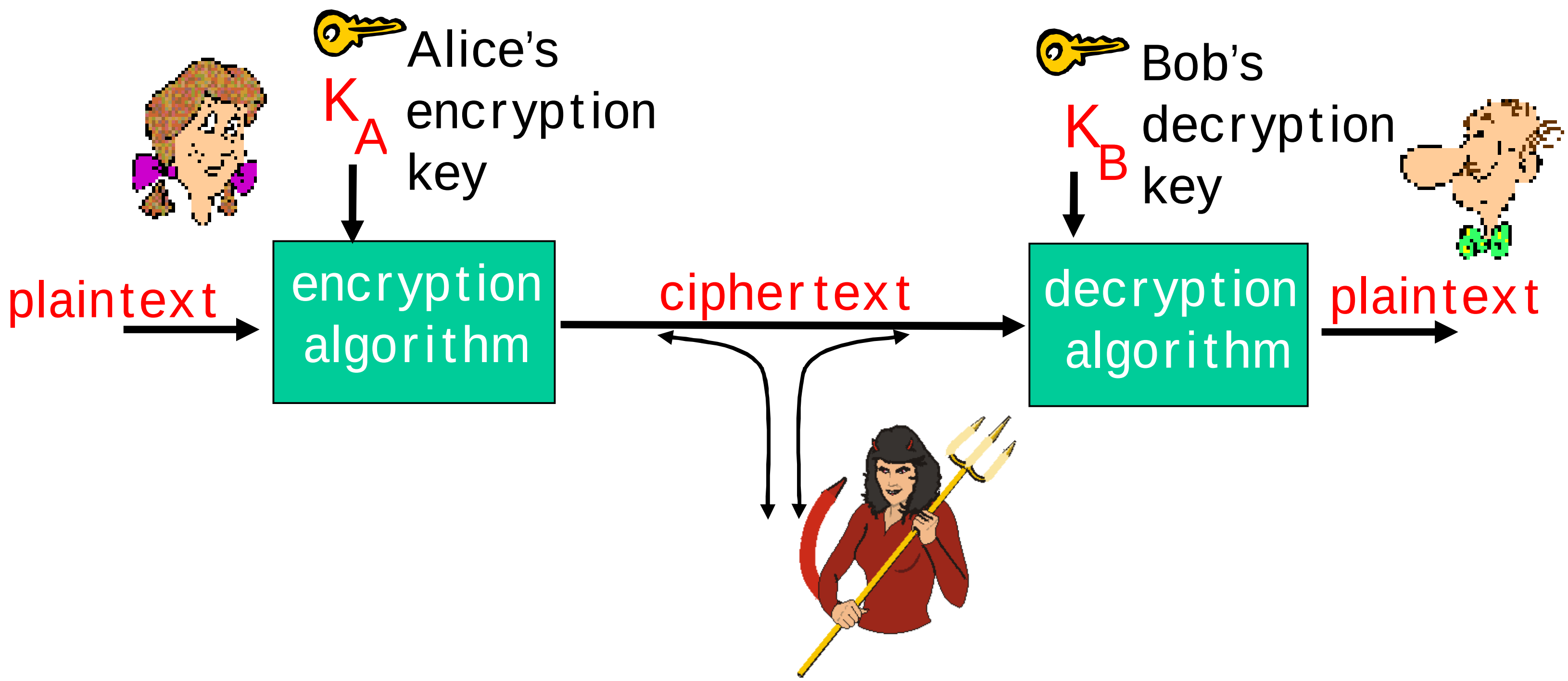
- well-known in network security world
- Bob, Alice (lovers!) want to communicate “securely”
- Trudy (intruder) may intercept, delete, add messages



Who might Bob, Alice be?

- ... well, *real-life* Bobs and Alices!
- Web browser/server for electronic transactions (e.g., on-line purchases)
- on-line banking client/server
- DNS servers
- routers exchanging routing table updates

The language of cryptography



m plaintext message

$K_A(m)$ ciphertext, encrypted with key K_A

$m = K_B(K_A(m))$

Simple encryption scheme

substitution cipher: substituting one thing for another

- monoalphabetic cipher: substitute one letter for another

```
 plaintext:  abcdefghijklmnopqrstuvwxyz

ciphertext:  mnbvcxzasdfghjklpoiuytrewq
```

E.g.:
```
 Plaintext: bob. i love you. alice
ciphertext: nkn. s gktc wky. mgsbc
```

Key: the mapping from the set of 26 letters to the set of 26 letters

Polyalphabetic encryption

- n monoalphabetic cyphers, $M_1, M_2, \ldots, M_n$
- Cycling pattern:
  - e.g., n=4, M_1, M_3, M_4, M_3, M_2; M_1, M_3, M_4, M_3, M_2;
- For each new plaintext symbol, use subsequent monoalphabetic pattern in cyclic pattern
  - dog: d from M_1, o from M_3, g from M_4
- Key: the n ciphers and the cyclic pattern

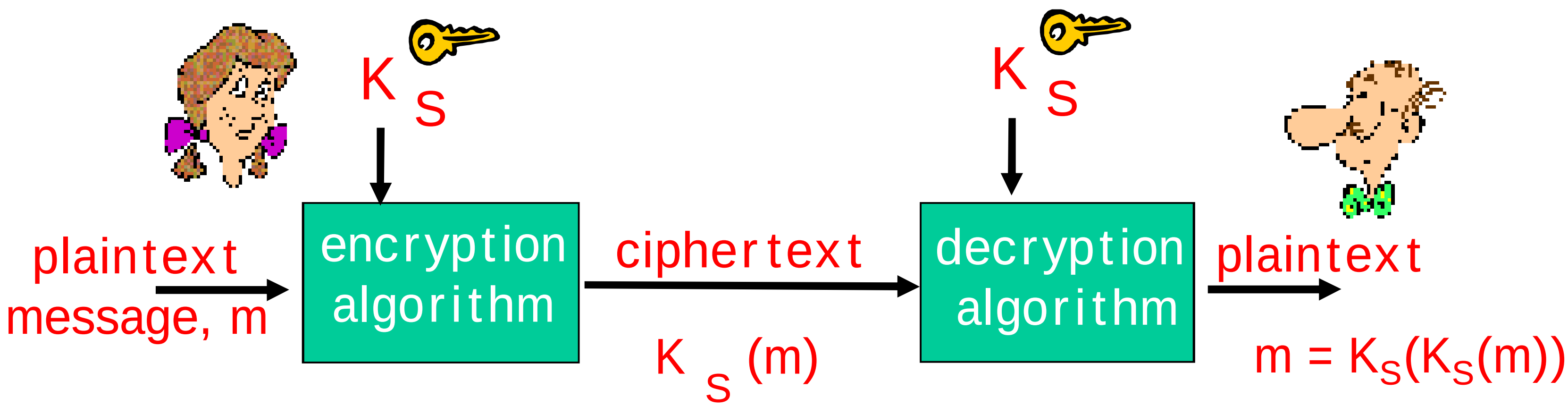
Breaking an encryption scheme

- Cipher-text only attack: Trudy has ciphertext that she can analyze
- Two approaches:
  - Search through all keys: must be able to differentiate resulting plaintext from gibberish
  - Statistical analysis
- Known-plaintext attack: trudy has some plaintext corresponding to some ciphertext
  - eg, in monoalphabetic cipher, trudy determines pairings for a,l,i,c,e,b,o,
- Chosen-plaintext attack: trudy can get the cyphertext for some chosen plaintext

Types of Cryptography

- Crypto often uses keys:
  - Algorithm is known to everyone
  - Only “keys” are secret
- Public key cryptography
  - Involves the use of two keys
- Symmetric key cryptography
  - Involves the use one key
- Hash functions
  - Involves the use of no keys
  - Nothing secret: How can this be useful?

Symmetric key cryptography



symmetric key crypto: Bob and Alice share same (symmetric) key: K_S

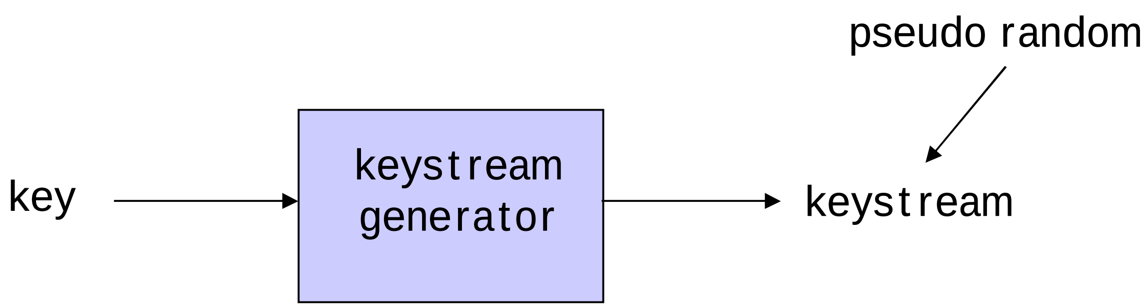
- e.g., key is knowing substitution pattern in mono alphabetic substitution cipher

Q: how do Bob and Alice agree on key value?

Two types of symmetric ciphers

- Stream ciphers
  - encrypt one bit at time
- Block ciphers
  - Break plaintext message in equal-size blocks
  - Encrypt each block as a unit

Stream Ciphers



- Combine each bit of keystream with bit of plaintext to get bit of ciphertext
- m(i) = ith bit of message
- ks(i) = ith bit of keystream
- c(i) = ith bit of ciphertext
- $c(i) = ks(i) \oplus m(i)$ ($\oplus$ = exclusive or)
- $m(i) = ks(i) \oplus c(i)$

Problems with stream ciphers

Known plain-text attack

- There's often predictable and repetitive data in communication messages
- attacker receives some cipher text c and correctly guesses corresponding plaintext m
- $ks = m \oplus c$
- Attacker now observes c', obtained with same sequence ks
- $m' = ks \oplus c'$

Even easier

- Attacker obtains two ciphertexts, c and c', generating with same key sequence
- $c \oplus c' = m \oplus m'$
- There are well known methods for decrypting 2 plaintexts given their XOR

Integrity problem too

- suppose attacker knows c and m (eg, plaintext attack);
- wants to change m to m'
- calculates $c' = c \oplus (m \oplus m')$
- sends c' to destination

RC4 Stream Cipher

- RC4 is a popular stream cipher
  - Extensively analyzed and considered good
  - Key can be from 1 to 256 bytes
  - Used in WEP for 802.11
  - Can be used in SSL

Block ciphers

- Message to be encrypted is processed in blocks of k bits (e.g., 64-bit blocks).
- 1-to-1 mapping is used to map k-bit block of plaintext to k-bit block of ciphertext

Example with k=3:

input	output
000	110
001	111
010	101
011	100

input	output
100	011
101	010
110	000
111	001

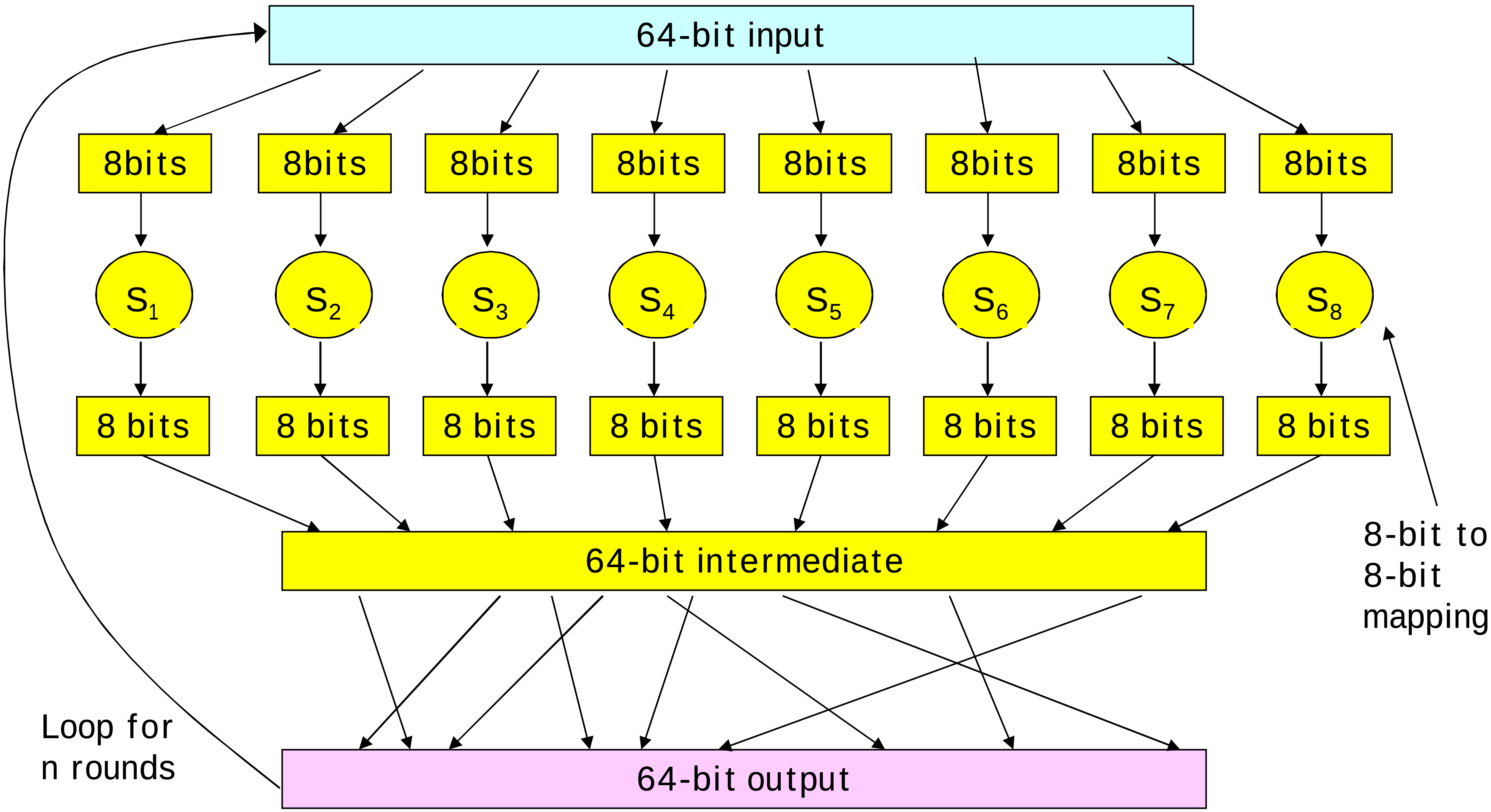
What is the ciphertext for 010110001111 ?

Block ciphers

- How many possible mappings are there for k=3?
  - How many 3-bit inputs?
  - How many permutations of the 3-bit inputs?
  - Answer: 40,320 ; not very many!
- In general, $2^k!$ mappings; huge for k=64
- Problem:
  - Table approach requires table with 2^{64} entries, each entry with 64 bits
- Table too big: instead use function that simulates a randomly permuted table

Prototype function

From Kaufman et al



Why rounds in prototpe?

- If only a single round, then one bit of input affects at most 8 bits of output.
- In 2nd round, the 8 affected bits get scattered and inputted into multiple substitution boxes.
- How many rounds?
  - How many times do you need to shuffle cards
  - Becomes less efficient as n increases

Encrypting a large message

- Why not just break message in 64-bit blocks, encrypt each block separately?
  - If same block of plaintext appears twice, will give same cyphertext.
- How about:
  - Generate random 64-bit number r(i) for each plaintext block m(i)
  - Calculate $c(i) = K_S(m(i) \oplus r(i))$
  - Transmit c(i), r(i), i=1,2,...
  - At receiver: $m(i) = K_S(c(i)) \oplus r(i)$
  - Problem: inefficient, need to send c(i) and r(i)

Cipher Block Chaining (CBC)

- CBC generates its own random numbers
  - Have encryption of current block depend on result of previous block
  - $c(i) = K_S(\ m(i) \oplus c(i-1)\ )$
  - $m(i) = K_S(\ c(i)) \oplus c(i-1)$
- How do we encrypt first block?
  - Initialization vector (IV): random block = c(0)
  - IV does not have to be secret
- Change IV for each message (or session)
  - Guarantees that even if the same message is sent repeatedly, the ciphertext will be completely different each time

Symmetric key crypto: DES

DES: Data Encryption Standard

- US encryption standard [NIST 1993]
- 56-bit symmetric key, 64-bit plaintext input
- Block cipher with cipher block chaining
- How secure is DES?
  - DES Challenge: 56-bit-key-encrypted phrase decrypted (brute force) in less than a day
  - No known good analytic attack
- making DES more secure:
  - 3DES: encrypt 3 times with 3 different keys

  (actually encrypt, decrypt, encrypt)

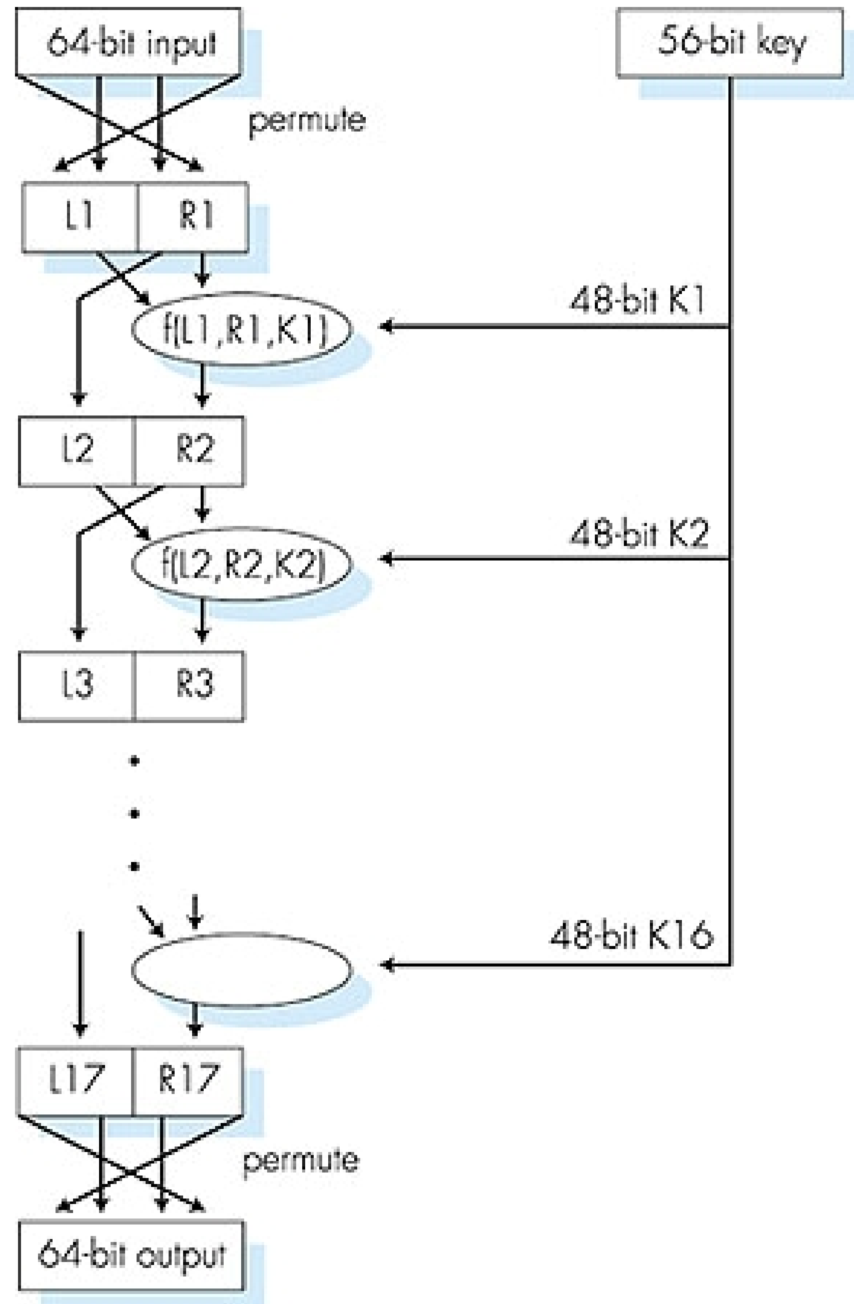
Symmetric key crypto: DES

DES operation

initial permutation

16 identical "rounds" of function application, each using different 48 bits of key

final permutation



AES: Advanced Encryption Standard

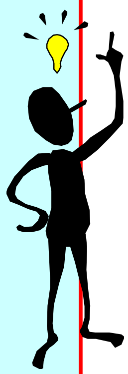
- new (Nov. 2001) symmetric-key NIST standard, replacing DES
- processes data in 128 bit blocks
- 128, 192, or 256 bit keys
- brute force decryption (try each key) taking 1 sec on DES, takes 149 trillion years for AES

Public Key Cryptography

symmetric key crypto

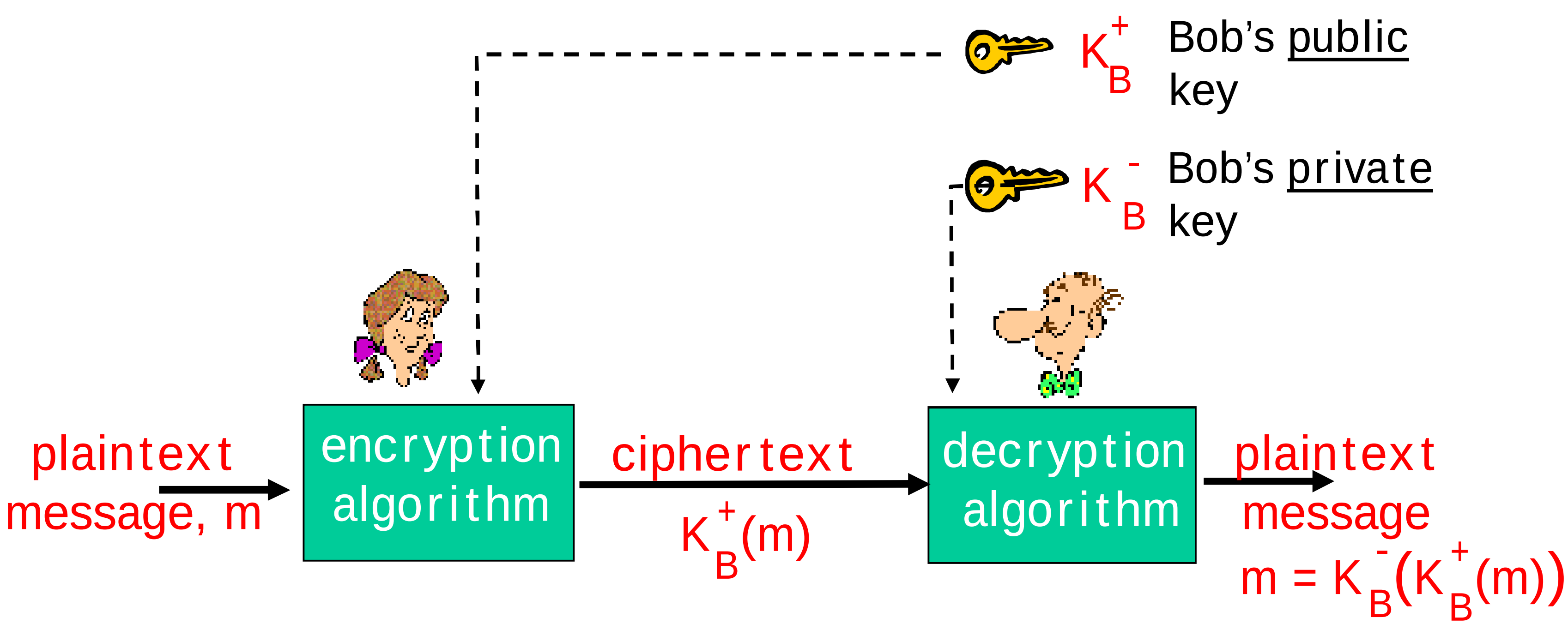
- requires sender, receiver know shared secret key
- Q: how to agree on key in first place (particularly if never "met")?

public key cryptography



- radically different approach [Diffie-Hellman76, RSA78]
- sender, receiver do *not* share secret key
- *public* encryption key known to *all*
- *private* decryption key known only to receiver

Public key cryptography



Public key encryption algorithms

Requirements:

1. need $K_B^+(\cdot)$ and $K_B^-(\cdot)$ such that

$$K_B^-(K_B^+(m)) = m$$

2. given public key K_B^+, it should be impossible to compute private key K_B^-

RSA: Rivest, Shamir, Adelson algorithm

Prerequisite: modular arithmetic

- x mod n = remainder of x when divide by n
- Facts:

  $[(a \bmod n) + (b \bmod n)] \bmod n = (a+b) \bmod n$

  $[(a \bmod n) - (b \bmod n)] \bmod n = (a-b) \bmod n$

  $[(a \bmod n) * (b \bmod n)] \bmod n = (a*b) \bmod n$
- Thus

  $(a \bmod n)^d \bmod n = a^d \bmod n$
- Example: x=14, n=10, d=2:

  $(x \bmod n)^d \bmod n = 4^2 \bmod 10 = 6$

  $x^d = 14^2 = 196 \quad x^d \bmod 10 = 6$

RSA: getting ready

- A message is a bit pattern.
- A bit pattern can be uniquely represented by an integer number.
- Thus encrypting a message is equivalent to encrypting a number.

Example

- m= 10010001 . This message is uniquely represented by the decimal number 145.
- To encrypt m, we encrypt the corresponding number, which gives a new number (the cyphertext).

RSA: Creating public/private key pair

1. Choose two large prime numbers $p, q.$
   (e.g., 1024 bits each)

2. Compute $n = pq$, $z = (p-1)(q-1)$

3. Choose e (with $e<n$) that has no common factors with z. (e, z are "relatively prime").

4. Choose d such that $ed-1$ is exactly divisible by z. (in other words: $ed \bmod z = 1$).

5. *Public* key is $\underbrace{(n,e)}_{K_B^+}$. *Private* key is $\underbrace{(n,d)}_{K_B^-}$.

RSA: Encryption, decryption

0. Given (n,e) and (n,d) as computed above

1. To encrypt message m $(<n)$, compute

   $c = m^e \bmod n$

2. To decrypt received bit pattern, c, compute

   $m = c^d \bmod n$

Magic happens! $m = (\underbrace{m^e \bmod n}_{c})^d \bmod n$

RSA example:

Bob chooses *p=5, q=7*. Then *n=35, z=24*.

e=5 (so *e, z* relatively prime).

d=29 (so *ed-1* exactly divisible by z).

Encrypting 8-bit messages.

	bit pattern	m	m^e	$c = m^e \bmod n$
encrypt:	0000l000	12	24832	17

	c	c^d	$m = c^d \bmod n$
decrypt:	17	481968572106750915091411825223071697	12

Why does RSA work?

- Must show that $c^d \bmod n = m$ where $c = m^e \bmod n$
- Fact: for any x and y: $x^y \bmod n = x^{(y \bmod z)} \bmod n$
  - where $n = pq$ and $z = (p-1)(q-1)$
- Thus,

$$
\begin{aligned}
c^d \bmod n &= (m^e \bmod n)^d \bmod n \\
&= m^{ed} \bmod n \\
&= m^{(ed \bmod z)} \bmod n \\
&= m^1 \bmod n \\
&= m
\end{aligned}
$$

RSA: another important property

The following property will be *very* useful later:

$$\underbrace{K_B^-(K_B^+(m))}_{\substack{\text{use public key}\\\text{first, followed}\\\text{by private key}}} = m = \underbrace{K_B^+(K_B^-(m))}_{\substack{\text{use private key}\\\text{first, followed}\\\text{by public key}}}$$

Result is the same!

Why $K_B^-(K_B^+(m)) = m = K_B^+(K_B^-(m))$?

Follows directly from modular arithmetic:

$$(m^e \bmod n)^d \bmod n = m^{ed} \bmod n$$
$$= m^{de} \bmod n$$
$$= (m^d \bmod n)^e \bmod n$$

Why is RSA Secure?

- Suppose you know Bob's public key (n,e). How hard is it to determine d?
- Essentially need to find factors of n without knowing the two factors p and q.
- Fact: factoring a big number is hard.

Generating RSA keys

- Have to find big primes p and q
- Approach: make good guess then apply testing rules (see Kaufman)

Session keys

- Exponentiation is computationally intensive
- DES is at least 100 times faster than RSA

Session key, K_S

- Bob and Alice use RSA to exchange a symmetric key K_S
- Once both have K_S, they use symmetric key cryptography

Diffie-Hellman

- Allows two entities to agree on shared key.
  - But does not provide encryption
- p is a large prime; g is a number less than p.
  - p and g are made public
- Alice and Bob each separately choose 512-bit random numbers, S_A and S_B.
  - the private keys
- Alice and Bob compute public keys:
  - $T_A = g^{S_A} \bmod p$; $T_B = g^{S_B} \bmod p$;

Diffie-Helman (2)

- Alice and Bob exchange T_A and T_B in the clear
- Alice computes $(T_B)^{S_A}$ mod p
- Bob computes $(T_A)^{S_B}$ mod p
- shared secret:
  - $S = (T_B)^{S_A} \bmod p == g^{S_A S_B} \bmod p = (T_A)^{S_B} \bmod p$
- Even though Trudy might sniff T_B and T_A, Trudy cannot easily determine S.
- Problem: Man-in-the-middle attack:
  - Alice doesn't know for sure that T_B came from Bob; may be Trudy instead
  - See Kaufman et al for solutions

Diffie-Hellman: Toy Example

- $p = 11$ and $g = 5$
- Private keys: $S_A = 3$ and $S_B = 4$

Public keys:

- $T_A = g^{S_A} \bmod p = 5^3 \bmod 11 = 125 \bmod 11 = 4$
- $T_B = g^{S_B} \bmod p = 5^4 \bmod 11 = 625 \bmod 11 = 9$

Exchange public keys & compute shared secret:

- $(T_B)^{S_A} \bmod p = 9^3 \bmod 11 = 729 \bmod 11 = 3$
- $(T_A)^{S_B} \bmod p = 4^4 \bmod 11 = 256 \bmod 11 = 3$

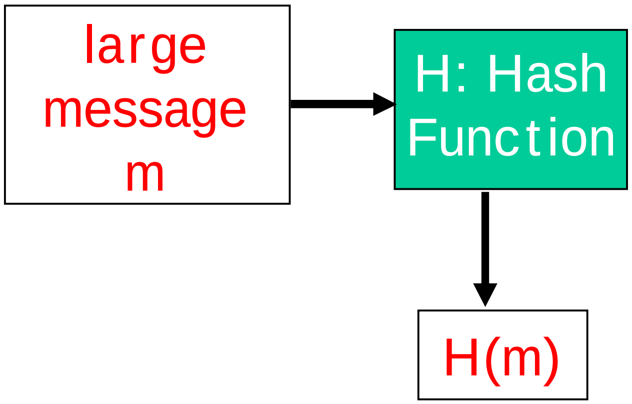
Shared secret:

- 3 = symmetric key

Message Integrity

- Allows communicating parties to verify that received messages are authentic.
  - Content of message has not been altered
  - Source of message is who/what you think it is
  - Message has not been artificially delayed (playback attack)
  - Sequence of messages is maintained
- Let's first talk about message digests

Message Digests



- Function H( ) that takes as input an arbitrary length message and outputs a fixed-length string: “message signature”
- Note that H( ) is a many-to-1 function
- H( ) is often called a “hash function”

- Desirable properties:
  - Easy to calculate
  - Irreversibility: Can’t determine m from H(m)
  - Collision resistance: Computationally difficult to produce m and m’ such that H(m) = H(m’)
  - Seemingly random output

Internet checksum: poor message digest

Internet checksum has some properties of hash function:

- ✓ produces fixed length digest (16-bit sum) of input
- ✓ is many-to-one

- ❒ But given message with given hash value, it is easy to find another message with same hash value.
- ❒ Example: Simplified checksum: add 4-byte chunks at a time:

message	ASCII format
I O U 1	**49 4F 55 31**
0 0 . 9	**30 30 2E 39**
9 B O B	**39 42 D2 42**
	B2 C1 D2 AC

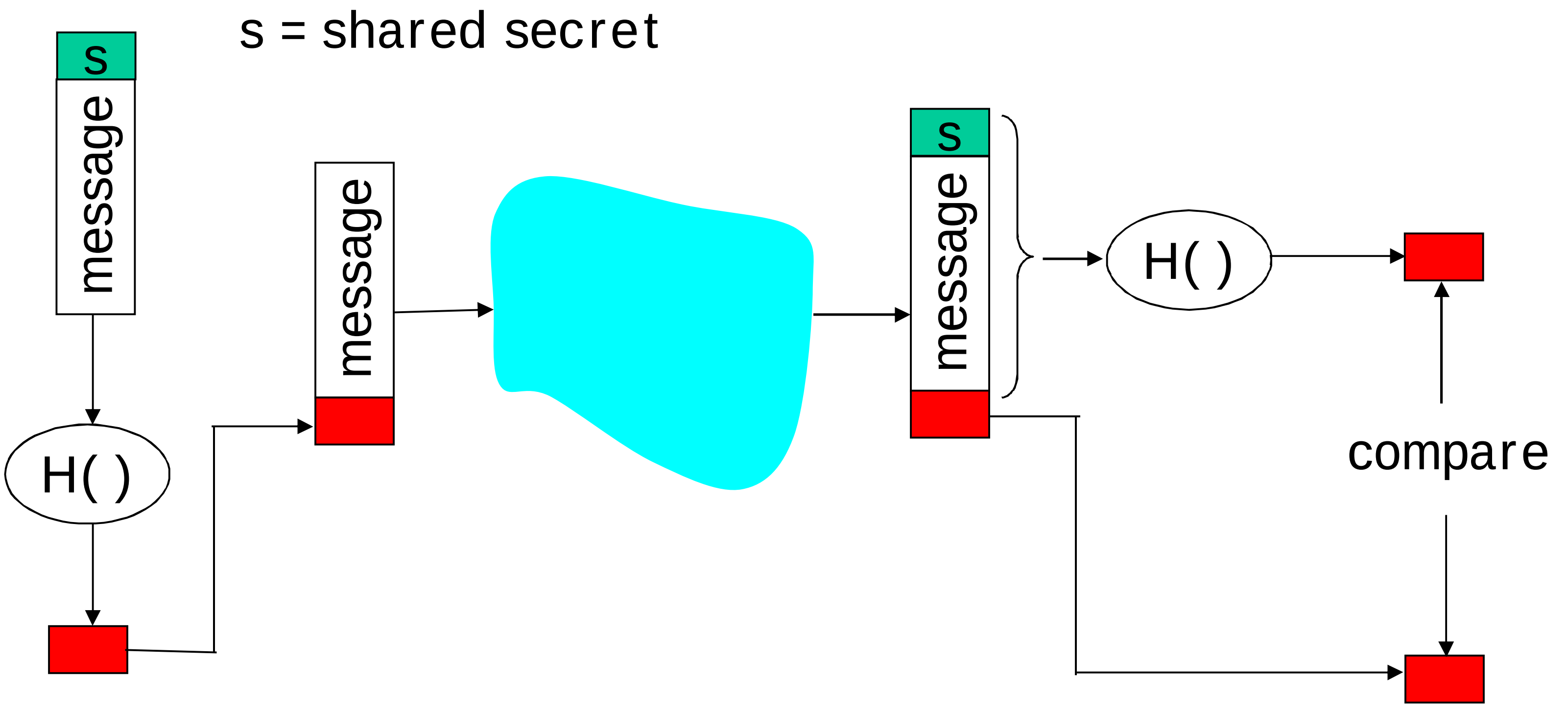
message	ASCII format
I O U 9	**49 4F 55 39**
0 0 . 1	**30 30 2E 31**
9 B O B	**39 42 D2 42**
	B2 C1 D2 AC

different messages but identical checksums!

Hash Function Algorithms

- MD5 hash function widely used (RFC 1321)
  - computes 128-bit message digest in 4-step process.
- SHA-1 is also used.
  - US standard [NIST, FIPS PUB 180-1]
  - 160-bit message digest

Message Authentication Code (MAC)



- ***Authenticates sender***
- ***Verifies message integrity***
- No encryption !
- Also called “keyed hash”
- Notation: $MD_m = H(s||m)$; send $m||MD_m$

HMAC

- Popular MAC standard
- Addresses some subtle security flaws

1. Concatenates secret to front of message.
2. Hashes concatenated message
3. Concatenates the secret to front of digest
4. Hashes the combination again.

Example: OSPF

- Recall that OSPF is an intra-AS routing protocol
- Each router creates map of entire AS (or area) and runs shortest path algorithm over map.
- Router receives link-state advertisements (LSAs) from all other routers in AS.

Attacks:

- Message insertion
- Message deletion
- Message modification

- How do we know if an OSPF message is authentic?

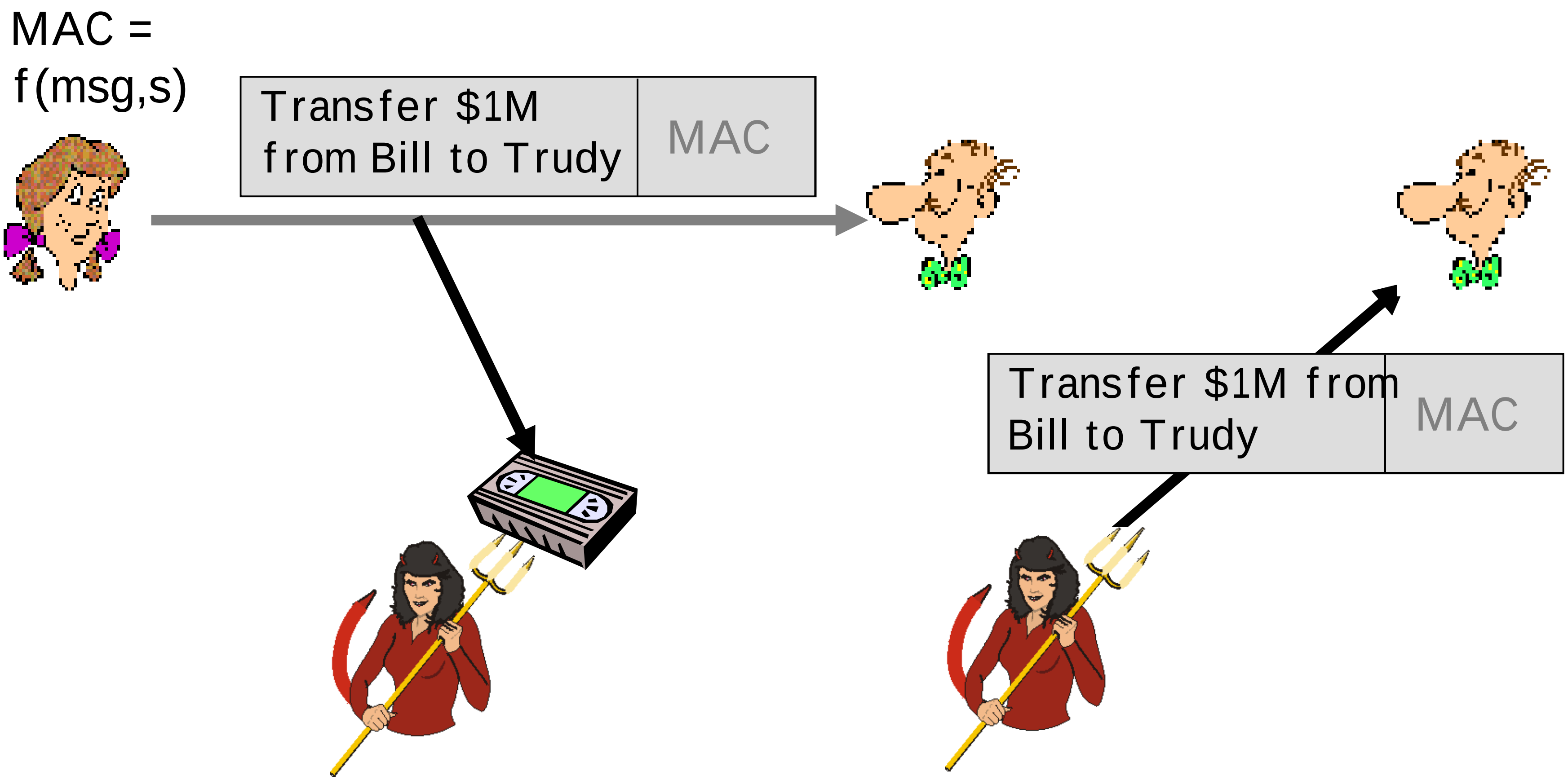
OSPF Authentication

- Within an Autonomous System, routers send OSPF messages to each other.
- OSPF provides authentication choices
  - No authentication
  - Shared password: inserted in clear in 64-bit authentication field in OSPF packet
  - Cryptographic hash
- Cryptographic hash with MD5
  - 64-bit authentication field includes 32-bit sequence number
  - MD5 is run over a concatenation of the OSPF packet and shared secret key
  - MD5 hash then appended to OSPF packet; encapsulated in IP datagram

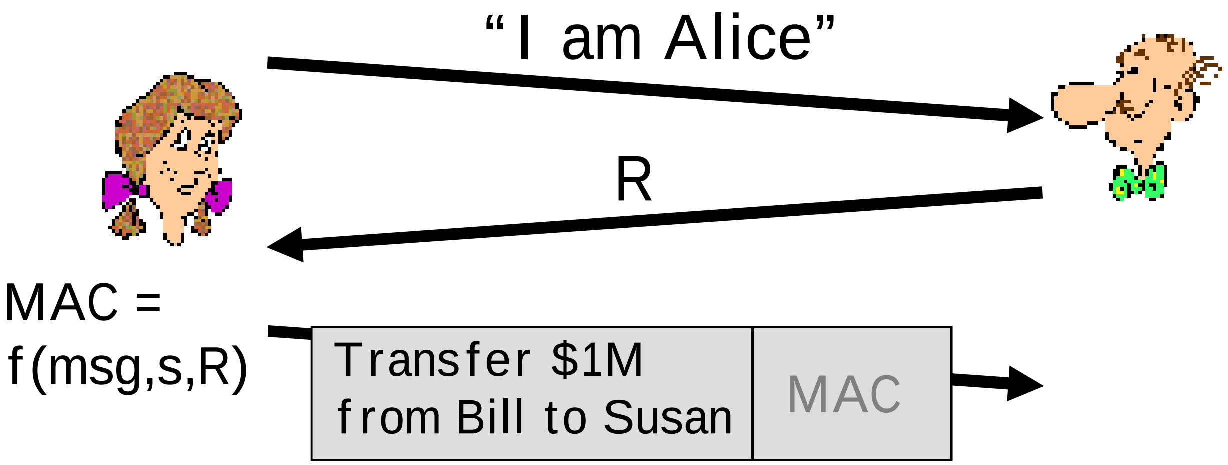
End-point authentication

- Want to be sure of the originator of the message – *end-point authentication*.
- Assuming Alice and Bob have a shared secret, will MAC provide message authentication.
  - We do know that Alice created the message.
  - But did she send it?

Playback attack



Defending against playback attack: nonce



Digital Signatures

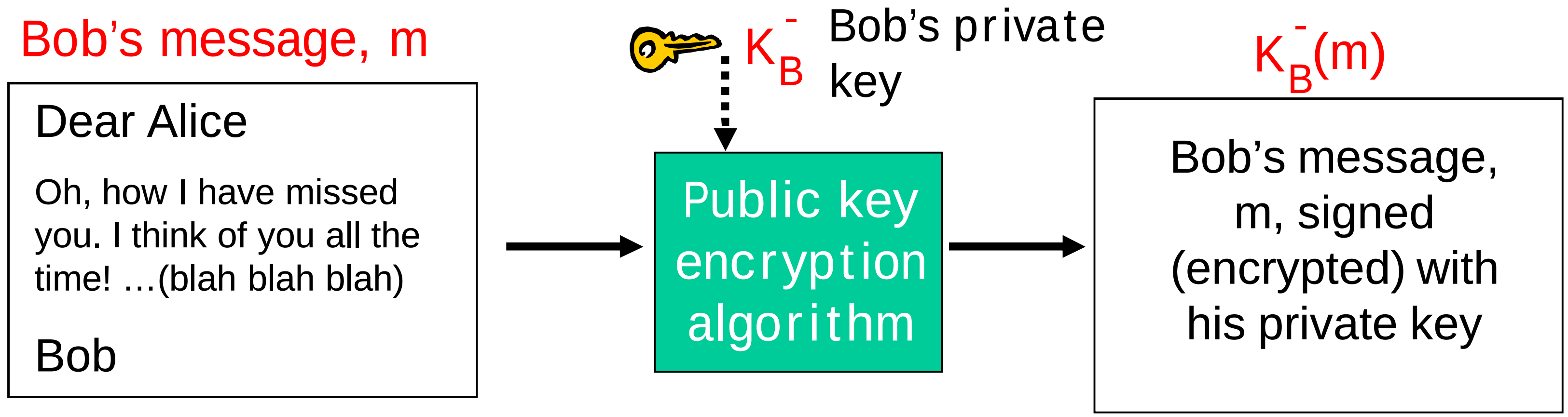
Cryptographic technique analogous to hand-written signatures.

- sender (Bob) digitally signs document, establishing he is document owner/creator.
- Goal is similar to that of a MAC, except now use public-key cryptography
- verifiable, nonforgeable: recipient (Alice) can prove to someone that Bob, and no one else (including Alice), must have signed document

Digital Signatures

Simple digital signature for message m:

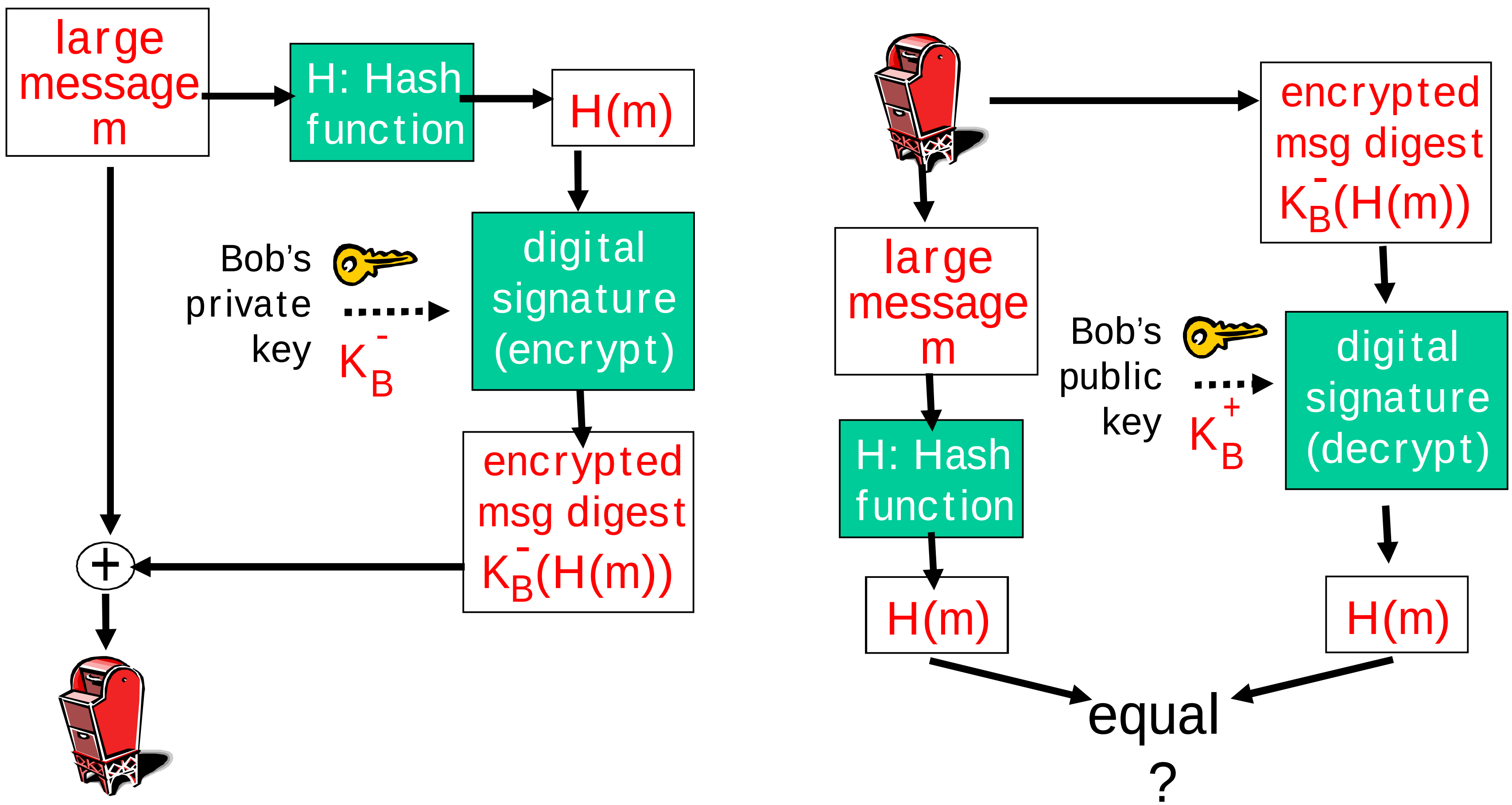
- Bob signs m by encrypting with his private key K_B^-, creating "signed" message, $K_B^-(m)$



Digital signature = signed message digest

Bob sends digitally signed message:

Alice verifies signature and integrity of digitally signed message:



Digital Signatures (more)

- Suppose Alice receives msg m, digital signature $K_B^-(m)$
- Alice verifies m signed by Bob by applying Bob's public key K_B^+ to $K_B^-(m)$ then checks $K_B^+(K_B^-(m)) = m$.
- If $K_B^+(K_B^-(m)) = m$, whoever signed m must have used Bob's private key.

Alice thus verifies that:

- ✓ Bob signed m.
- ✓ No one else signed m.
- ✓ Bob signed m and not m'.

Non-repudiation:

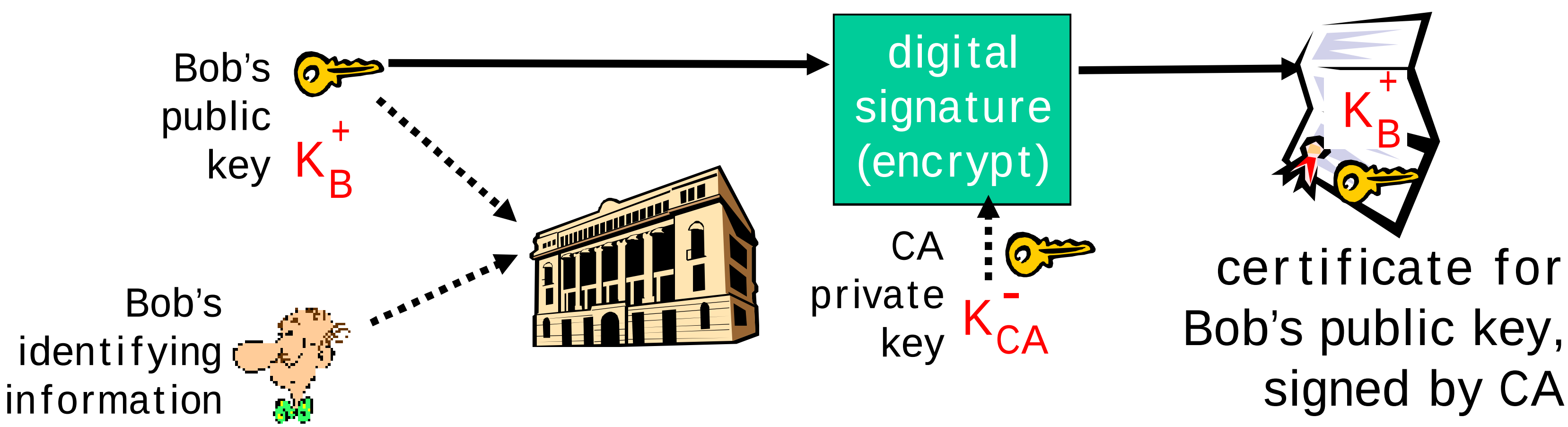
- ✓ Alice can take m, and signature $K_B^-(m)$ to court and prove that Bob signed m.

Public-key certification

- Motivation: Trudy plays pizza prank on Bob
  - Trudy creates e-mail order:
    Dear Pizza Store, Please deliver to me four pepperoni pizzas. Thank you, Bob
  - Trudy signs order with her private key
  - Trudy sends order to Pizza Store
  - Trudy sends to Pizza Store her public key, but says it's Bob's public key.
  - Pizza Store verifies signature; then delivers four pizzas to Bob.
  - Bob doesn't even like Pepperoni

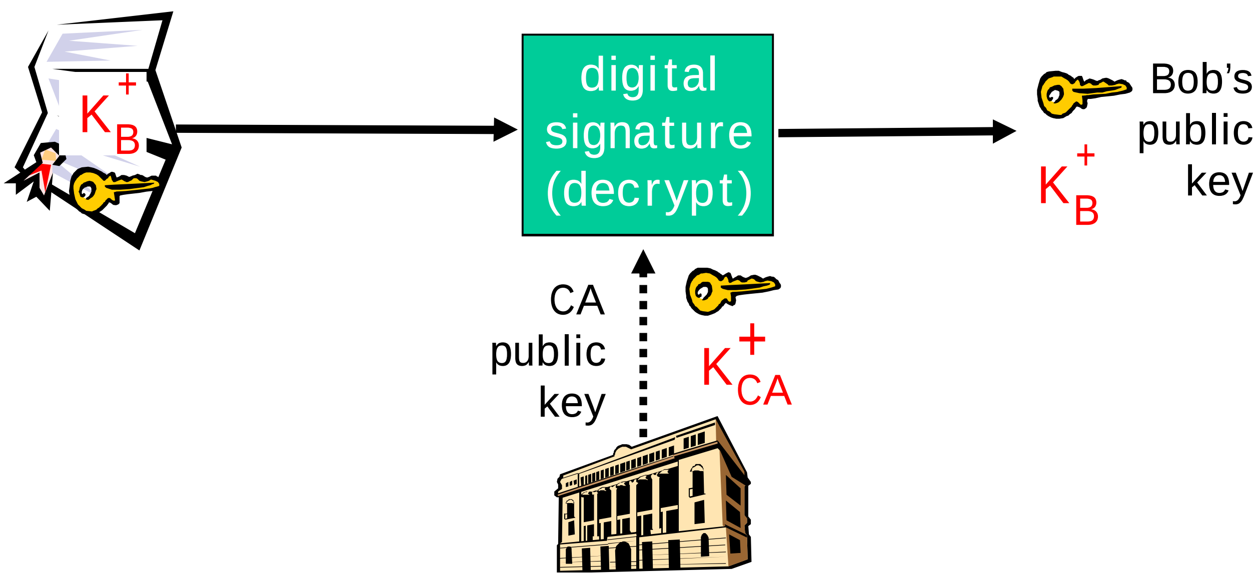
Certification Authorities

- Certification authority (CA): binds public key to particular entity, E.
- E (person, router) registers its public key with CA.
  - E provides "proof of identity" to CA.
  - CA creates certificate binding E to its public key.
  - certificate containing E's public key digitally signed by CA – CA says "this is E's public key"



Certification Authorities

- When Alice wants Bob's public key:
  - gets Bob's certificate (Bob or elsewhere).
  - apply CA's public key to Bob's certificate, get Bob's public key



Certificates: summary

- Primary standard X.509 (RFC 2459)
- Certificate contains:
  - Issuer name
  - Entity name, address, domain name, etc.
  - Entity's public key
  - Digital signature (signed with issuer's private key)
- Public-Key Infrastructure (PKI)
  - Certificates and certification authorities
  - Often considered "heavy"